

Part A (AB or BC): Graphing calculator required
Scoring Guidelines for Question 1

9 points

Learning Objectives: CHA-2.D CHA-3.A CHA-3.C CHA-3.F CHA-4.B LIM-5.A

- (a) Use the data in the table to approximate $R'(5)$. Show the computations that lead to your answer. Using correct units, explain the meaning of $R'(5)$ in the context of the problem.

Model Solution	Scoring	
$R'(5) = \frac{R(6) - R(4)}{6 - 4} = \frac{3010 - 3442}{2} = -216$	Approximation using values from table	1 point 2.B
At time $t = 5$ hours (12 P.M.), the rate at which vehicles cross the bridge is decreasing at a rate of approximately 216 vehicles per hour per hour.	Interpretation with units	1 point 3.F 4.B
Total for part (a)		2 points

- (b) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\int_0^{12} R(t) dt$. Indicate units of measure.

$\int_0^{12} R(t) dt = 4(R(2) + R(6) + R(10))$	Midpoint sum setup	1 point 1.E
$= 4(3653 + 3010 + 1986)$	Approximation using values from the table with units	1 point 2.B 4.B
$= 34,596 \text{ vehicles}$		
Total for part (b)		2 points

- (c) What is the average number of vehicles crossing the bridge per hour on the weekend day for $0 \leq t \leq 12$?

$\frac{1}{12 - 0} \int_0^{12} H(t) dt = 2452$	Definite integral	1 point 1.D 4.C
Definite integral	Answer with supporting work	1 point 1.B
Answer		
Total for part (c)		2 points

- (d) Use $L(t)$ to find the time t , for $12 \leq t \leq 17$, at which the rate of vehicles crossing the bridge is 2000 vehicles per hour. Show the work that leads to your answer.

$L(t) = H(12) - H'(12)(t - 12)$	Slope	1 point 1.B 4.E
$H(12) = 2596; H'(12) = -216$		
$L(t) = 2000$	$L(t) = 2000$	1 point 1.D
$\Rightarrow t = 14.759$	Answer with supporting work	1 point 1.E 4.E
Total for part (d)		3 points

Total for Question 1 **9 points**

Part A (AB or BC): Calculator not Permitted
Scoring Guidelines for Question 2

9 points

Learning Objectives: FUN-3.B FUN-4.A FUN-5.A FUN-6.D

- (a) On what open intervals contained in $(0,4)$ is the graph of f both decreasing and concave down?
 Give a reason for your answer.

Model Solution	Scoring	
The graph of f is decreasing and concave down on the intervals $(1, 1.6)$ and $(3, 3.5)$	Answer	1 point 2.E
because f' is negative and decreasing on these intervals.	Reason	1 point 3.E 4.A
Total for part (a)		2 points

- (b) Find the absolute minimum value of f on the interval $[0, 4]$. Justify your answer.

The graph of f' changes from negative to positive only at $x = 2$.	Considers $x = 2$ as a candidate	1 point 3.B
$f(0) = f(2) + \int_2^0 f'(x) dx = f(2) - \int_0^2 f'(x) dx = 5 - (2 - 6) = 9$ $f(2) = 5$ $f(4) = f(2) + \int_2^4 f'(x) dx = 5 + (10 - 14) = 1$	Answer with justification	1 point 3.E
On the interval $[0, 4]$, the absolute minimum value of f is $f(4) = 1$.		
Total for part (b)		2 points

- (c) Evaluate $\int_0^4 f(x)f'(x) dx$.

$\int_0^4 f(x)f'(x) dx = \frac{1}{2}(f(x))^2 \Big _{x=0}^{x=4}$ $= \frac{1}{2}((f(4))^2 - (f(0))^2)$ $= \frac{1}{2}(1^2 - 9^2) = -40$	Antiderivative of the form $a[f(x)]^2$	1 point 1.C
	Earned the first point and $a = \frac{1}{2}$	1 point 1.E
	Answer	1 point 2.B
Total for part (c)		3 points

- (d) Find $g'(2)$. Show the work that leads to your answer.

$g'(x) = 3x^2f(x) + x^3f'(x)$ $g'(2) = 3 \cdot 2^2f(2) + 2^3f'(2) = 12 \cdot 5 + 8 \cdot 0 = 60$	Product Rule	1 point 1.E
	Answer	1 point 2.B
Total for part (d)		2 points

Total for Question 2 **9 points**

Section I: Multiple Choice**45 Questions | 1 Hour 45 minutes | 50% of Exam Score**

- Part A: 30 questions; 60 minutes (calculator not permitted).
- Part B: 15 questions; 45 minutes (graphing calculator required).
- Questions include algebraic, exponential, logarithmic, trigonometric, and general types of functions.
- Questions include analytical, graphical, tabular, and verbal types of representations.

Section II: Free Response**6 Questions | 1 Hour 30 Minutes | 50% of Exam Score**

- Part A: 2 questions; 30 minutes (graphing calculator required).
- Part B: 4 questions; 60 minutes (calculator not permitted).
- Questions include various types of functions and function representations and a roughly equal mix of procedural and conceptual tasks.
- Questions include at least 2 questions that incorporate a real-world context or scenario into the question.

20011B (calc)

1. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S , where $S(t)$ is measured in millimeters and t is measured in days for $0 \leq t \leq 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2 \sin(0.03t) + 1.5$.
- (d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M , where $M(t) = \frac{1}{400}(3t^3 - 30t^2 + 330t)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \leq t \leq 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \leq t \leq 60$ to justify that there exists a time t , $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

$$\text{Rate of Monsoon} = M'(t) = \frac{1}{400}(9t^2 - 60t + 330)$$

$$\text{Rate of Storm} = S'(t) = 2 \sin(0.03t) + 1.5$$

$$M'(t) = S'(t)$$

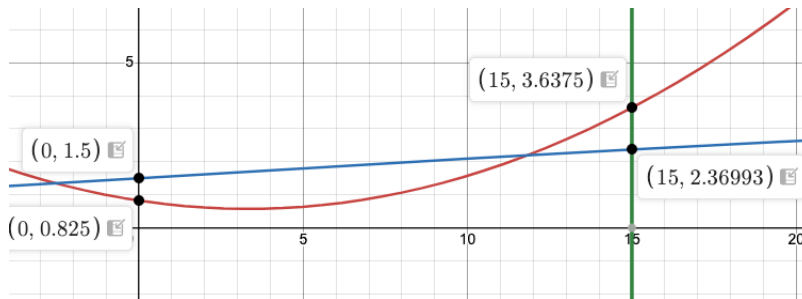
$$M'(t) - S'(t) = 0 \text{ For Same Rate}$$

$$M'(0) = \frac{330}{400} = .685$$

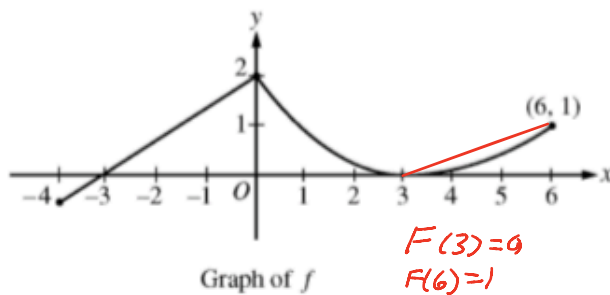
$$S'(0) = 1.5$$

$$M'(0) - S'(0) = .685 - 1.5 = -0.815 \rightarrow \text{MUST PASS}$$

$$M'(60) - S'(60) = 72.825 - 3.4477 = 69.3775 \rightarrow \text{I.V.T}$$



2009B



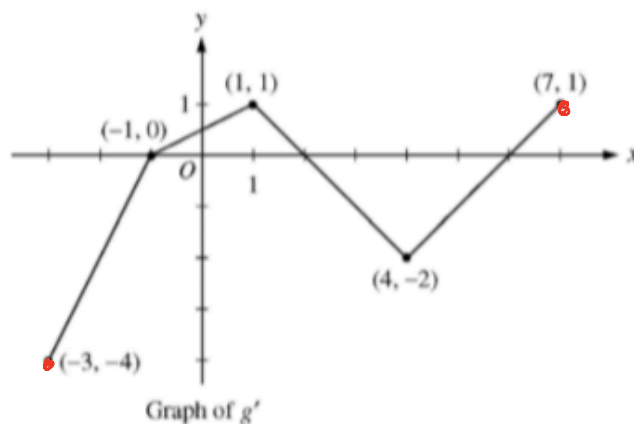
$$\frac{F(3) - F(6)}{3 - 6} = \frac{0 - 1}{-3} = \frac{1}{3}$$

3. A continuous function f is defined on the closed interval $-4 \leq x \leq 6$. The graph of f consists of a line segment and a curve that is tangent to the x -axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$. CONCAVE UP CONTINUOUS
- (c) Is there a value of a , $-4 \leq a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c , $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

DIFF $[3, 6]$

u

2008B



5. Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g' , the derivative of g , is shown above for $-3 \leq x \leq 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \leq x \leq 7$. Does the Mean Value Theorem applied on the interval $-3 \leq x \leq 7$ guarantee a value of c , for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

$$\frac{F(-3) - F(7)}{-3 - 7} = \frac{-4 - 1}{-10} = \frac{-5}{-10} = \frac{1}{2}$$

Nope Not Diff
 $g'(x) \uparrow$ at $x = -1, 1, 4$

$$g''(-1) = g''(1) = g''(4) = \phi$$

Sharp Points

2016

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

$R(t)$ is continuous

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(d) For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

Same $W(t) = R(t)$

$$W(t) - R(t) = 0$$

IVT

$$W(0) - R(0) = 2000 - 1340 = 660 \quad \text{got to pass}$$

$$W(8) - R(8) = 81.524 - 700 = -618.476$$

both $R(t)$ and $W(t)$ are continuous

2007

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	<u>2</u>	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x . The function h is given by $h(x) = f(g(x)) - 6$.
- (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.
- (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.
- (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t) dt$. Find the value of $w'(3)$.
- (d) If g^{-1} is the inverse function of g , write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

(a) $h(1) = F(g(1)) - 6 = F(2) - 6 = 9 - 6 = 3$
 $h(3) = F(g(3)) - 6 = F(4) - 6 = -1 - 6 = -7$ } gotta pass -5
 IVT
 h is continuous

(b) $h'(x) = F'(g(x)) \cdot g'(x)$
 $h'(1) = F'(g(1)) \cdot g'(1) = F'(2) \cdot g'(1) = 2 \cdot 5 = 10$
 $h'(3) = F'(g(3)) \cdot g'(3) = F'(4) \cdot g'(3) = 3 \cdot 2 = 6$ } using IVT won't work
 $\frac{h(1) - h(3)}{1 - 3} = \frac{3 - (-7)}{-2} = \frac{10}{-2} = -5$ MVT h is diff
 $h'(c) = -5 \quad 1 \leq c \leq 3$

Find $w'(3)$
 $w(x) = \int_1^{g(x)} f(t) dt$
 $w'(x) = F(g(x)) \cdot g'(x)$
 $w'(3) = F(g(3)) \cdot g'(3) = F(4) \cdot g'(3) = -1 \cdot 2 = -2$

(d)

(d) If g^{-1} is the inverse function of g , write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

$$(g^{-1})'(x) = \frac{1}{g'(g^{-1}(x))}$$

$$(g^{-1})'(2) = \frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(1)} = \frac{1}{5} = m$$

$$g^{-1}(2) = ? = 1 \quad (2, 1)$$

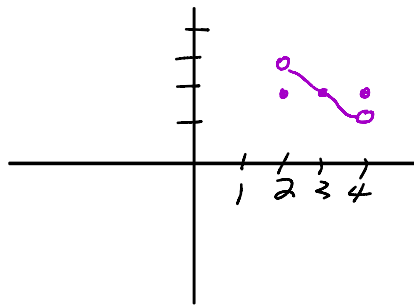
$$g(?) = 2$$

$$g(1) = 2 \quad (1, 2)$$

$$y - 1 = \frac{1}{5}(x - 2)$$

9. The function f is defined on the closed interval $[2, 4]$ and $f(2) = f(3) = f(4)$. On the open interval $(2, 4)$, f is continuous and strictly decreasing. Which of the following statements is true?

- (A) f attains neither a minimum value nor a maximum value on the closed interval $[2, 4]$.
- (B) f attains a minimum value but does not attain a maximum value on the closed interval $[2, 4]$.
- (C) f attains a maximum value but does not attain a minimum value on the closed interval $[2, 4]$.
- (D) f attains both a minimum value and a maximum value on the closed interval $[2, 4]$.



2. If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c ?

(A) $\frac{2\pi}{3}$

(B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{6}$

(D) π

(E) $\frac{3\pi}{2}$

$$\sin \frac{\pi}{2} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\sin \frac{3\pi}{2} = \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$$

$$MVT = \frac{\sin \frac{\pi}{4} - \sin \frac{3\pi}{4}}{\frac{\pi}{2} - \frac{3\pi}{2}} = \frac{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}}{-\pi} = 0$$

$$f'(x) = \cos\left(\frac{x}{2}\right) \cdot \frac{1}{2}$$

$$\frac{1}{2} \cos\left(\frac{x}{2}\right) = 0$$

$$\begin{aligned} \cos \frac{x}{2} &= 0 \\ \cos \frac{\pi}{2} &= 0 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} x = \pi$$

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$